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ABSTRACT

Research reported primarily from 1970 through 1975 on the use of manipulative materials at the elementary level (K-7) is reviewed. The research is categorized as design-oriented research, content-oriented research, and catalogs of manipulatives. Each section contains a summary followed by abstracts for each study. References to the theoretical foundation and historical background on the uses of materials, discussions of advantages and disadvantages, and comparisons of specific materials are also included. (MS)

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Literature Review:
Research on the Use of
Manipulatives in Mathematics
Learning

Max Gerling and Stewart Wood



1976

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PREFACE

An important objective of the Project for the Mathematical Development of Children was to investigate how children learn and think about mathematics. The use of manipulative aids plays an important role in this process. This publication is a review of recent research on the use of manipulatives in the teaching of mathematics in the elementary grades.

Many thanks to Merlyn Behr for his suggestions, guidance, and patience. Thanks are also due to the project administrative assistant, Janelle Hardy, for coordinating the technical aspects of the preparation of the report; and, to Mary Harrington and Joe Schmerler for the typing.

FOREWORD

Ed Begle recently remarked that curricular efforts during the 1960's taught us a great deal about how to teach better mathematics, but very little about how to teach mathematics better. The mathematician will, quite likely, agree with both parts of this statement. The layman, the parent, and the elementary school teacher, however, question the thesis that the "new math" was really better than the "old math." At best, the fruits of the mathematics curriculum "revolution" were not sweet. Many judge them to be bitter.

While some viewed the curricular changes of the 1960's to be "revolutionary," others disagreed. Thomas C. O'Brien of Southern Illinois University at Edwardsville recently wrote, "We have not made any fundamental change in school mathematics."¹ He cites Allendoerfer who suggested that a curriculum which heeds the ways in which young children learn mathematics is needed. Such a curriculum would be based on the understanding of children's thinking and learning. It is one thing, however, to recognize that a conceptual model for mathematics curriculum is sound and necessary and to ask that the child's thinking and learning processes be heeded; it is quite another to translate these ideas into a curriculum which can be used effectively by the ordinary elementary school teacher working in the ordinary elementary school classroom.

Moreover, to propose that children's thinking processes should serve as a basis for curriculum development is to presuppose that curriculum makers agree on what these processes are. Such is not the case, but even if it were, curriculum makers do not agree on the implications which the understanding of these thinking processes would have for curriculum development.

In the real world of today's elementary school classroom, where not much hope for drastic changes for the better can be foreseen, it appears that in order to build a realistic, yet sound basis for the mathematics curriculum, children's mathematical thinking must be studied intensively in their usual school habitat. Given an opportunity to think freely, children clearly display certain patterns of thought as they deal with ordinary mathematical situations encountered daily in their classroom. A videotaped record of the outward manifestations of a child's thinking, uninfluenced by any teaching on the part of the interviewer, provides a rich source for conjectures as to what this thinking is, what mental structures the child has developed, and how the child uses these structures when dealing with the ordinary concepts of arithmetic. In addition, an intensive analysis of this videotape generates some conjectures as to the possible sources of what adults view as children's "misconceptions" and about how the school environment (the teacher and the materials) "fights" the child's natural thought processes.

The Project for the Mathematical Development of Children (PMDC)² set out

¹"Why Teach Mathematics?" The Elementary School Journal 73 (Feb. 1973), 258-68.

²PMDC is supported by the National Science Foundation, Grant No. PES 74-18106-A03.

to create a more extensive and reliable basis on which to build mathematics curriculum. Accordingly, the emphasis in the first phase is to try to understand the children's intellectual pursuits, specifically their attempts to acquire some basic mathematical skills and concepts.

The PMDC, in its initial phase, works with children in grades 1 and 2. These grades seem to comprise the crucial years for the development of bases for the future learning of mathematics, since key mathematical concepts begin to form at these grade levels. The children's mathematical development is studied by means of:

1. One-to-one videotaped interviews subsequently analyzed by various individuals.
2. Teaching experiments in which specific variables are observed in a group teaching setting with five to fourteen children.
3. Intensive observations of children in their regular classroom setting.
4. Studies designed to investigate intensively the effect of a particular variable or medium on communicating mathematics to young children.
5. Formal testing, both group and one-to-one, designed to provide further insights into young children's mathematical knowledge.

The PMDC staff and the Advisory Board wish to report the Project's activities and findings to all who are interested in mathematical education. One means for accomplishing this is the PMDC publication program.

Many individuals contributed to the activities of PMDC. Its Advisory Board members are: Edward Begle, Edgar Edwards, Walter Dick, Renee Henry, John LeBlanc, Gerald Rising, Charles Smock, Stephen Willoughby and Lauren Woodby. The principal investigators are: Merlyn Behr, Tom Denmark, Stanley Erlwanger, Janice Flake, Larry Hatfield, William McKillip, Eugene D. Nichols, Leonard Pikaart, Leslie Steffe, and the Evaluator, Ray Carry. A special recognition for this publication is given to the PMDC Publications Committee, consisting of Merlyn Behr (Chairman), Thomas Cooney and Tom Denmark.

Eugene D. Nichols
Director of PMDC

INTRODUCTION

Increasing interest in the use of manipulatives in the teaching of mathematics has been displayed during the past ten years as evidenced by increasing research activity in this area. Of necessity the scope of this survey has been limited to include most studies reported from 1970 through 1975, which have focused on the role of manipulatives in the instruction of mathematics at the elementary level (K-7). Also included are some studies reported prior to 1970 and some studies involving older subjects, particularly those relating to elementary teacher training. Those studies of the use of manipulatives at the secondary level, as well as those centering on a more general "laboratory approach" to mathematics instruction have been omitted.

Discussions of the theoretical foundation for and historical background of the use of manipulative aids in the teaching of mathematics have been done very well by others and need not be repeated here. However, these references -- Beougher (1967), Bruner (1960, 1964), Bruner and Kenny (1965), Burno and Davis (1970), Dienes (1959, 1961, 1963, 1967, 1969), Good (1971), Kieven (1969, 1970), Smedslund (1964), and Stern (1949) -- will be included in the bibliography. Also included in the bibliography are: (1) articles of interest which discuss the advantages and disadvantages of various manipulatives--Andrews and Nelson (1963), Nasca (1966), Reys (1971, 1972), Sowell (1974), and Suydarn and Weaver (1970); (2) studies evaluating and/or comparing the Cuisenaire, Dienes, Sterns, and traditional methods--Brownell (1963, 1964, 1968), Hollis (1965), Lucow (1963, 1964), Passey (1963), and Williams (1963, 1972); and (3) a description of the Nuffield Project and the emphasis it places on a concrete approach to teaching mathematics--Keane (1973).

Descriptions of recent research on the use of manipulatives are found in the three parts of this report: I. Design-oriented research; II. Content-oriented research; and, III. Catalogs of manipulatives.

I. Design-oriented Research

Much of the research on manipulatives was what we termed design-oriented; that is, it was not centered on the use of manipulatives for a particular topic or grade level, but was broader in scope, covering perhaps multiple topics and grade levels. This research is organized under the following seven headings: studies which (a) compared the use of a manipulative with no manipulative, (b) compared the uses of several manipulatives, (c) tested Dienes multi-embodiment hypothesis, (d) compared enactive, iconic, and symbolic modes of presentation, (3) compared different kinds of classroom use for a manipulative, (f) investigated the effect of using manipulatives on attitude, and (g) investigated differential effects of the use of manipulatives with students having various learner characteristics.

(a) Among the studies which compared the use of a manipulative with no manipulative were those typically described by the experimenters as manipulative versus non-manipulative approaches, multi-sensory versus textbook approach, concrete versus textbook approach, concrete versus symbolic approach, concrete and abstract teaching, activity approach and lecture approach, tangible versus routine presentation, and manipulative versus paper and pencil activities.

In a study with five-year-olds, Churchill (1958) found that children who had an opportunity to play with materials developed mathematical concepts more quickly than those who did not. Clausen (1971), in a study with kindergarten and first grade pupils, found no significant differences in achievement, but did find a trend toward higher achievement among pupils exposed to a multi-sensory approach. Weber (1969), investigating a manipulative versus a paper and pencil approach with first graders, also found no significant differences in achievement, but on an experimenter made oral test of understanding did find that children from the manipulative treatment scored significantly higher in correct responses and levels of understanding.

Fennema (1972), in a study with second grade children, found that the efficacy of using a manipulative model in instruction depends less on the age of the learner than on his experiential background. Although both groups (symbolic and manipulative) performed equally well on direct recall tests, the symbolic treatment group scored significantly higher than the manipulative treatment group on transfer tests involving products between 11 and 16. Fennema attributes this in part to the presymbolic experiences of the children in the study which suggest that these children, with the appropriate prerequisite experiences, were ready to use the symbolic model, with its greater generalizability, more effectively than the manipulative model.

Davidson (1972), in a study comparing a concrete material approach with a conventional textbook approach with third and fourth graders, found no significant differences in performances on conservation tests with children in the two approaches and like IQ groups. Trask (1972) found that third grade pupils of above average computational ability were

helped more by a manipulative approach, while pupils of below average computational ability benefited more from a symbolic approach.

In a study with fourth, fifth, and sixth graders, Wallace (1974) found achievement of pupils in the manipulative approach significantly higher than that of students in the traditional approach. Spross (1962), in a study comparing a tangible with a routine approach, found that fifth and sixth graders scored significantly higher on reasoning items but not on fundamentals. Branch (1973) found that sixth graders taught using a manipulative approach scored significantly higher on immediate retention than those taught without manipulatives. For low ability seventh graders, Kuhfittig (1972) found that an approach using concrete aids produced significantly higher achievement scores, but no significant differences on retention tests.

A number of studies were also conducted with college level students, comparing activity approaches with conventional lecture-textbook approaches. Attitudinal differences were found and will be discussed later. C. W. Smith (1975) found that a manipulative approach produced significantly higher achievement than the conventional lecture approach, while G. J. Smith (1974), Turek (1972), and Weisman (1972) all found no significant differences in achievement gains. Warkentin (1975), however, found that a lecture approach produced significantly higher scores than a manipulative approach on a comprehensive final exam, but the manipulative groups were not able to cover as much material as the lecture groups.

(b) Some studies compared the uses of several manipulatives. Reddell and DeVault (1960) compared the effectiveness of three types of aids in improving understanding and achievement of fifth grade pupils and their teachers. Pupils in the two groups using commercially available aids made greater gains in achievement than pupils in the group using teacher-made aids. Significantly greater gains in understanding were also made by teachers of these two groups.

Harshman, Wells, and Payne (1962) compared the effectiveness of three different types of aids with first graders. They found no significant differences in class means, but using individual scores they found that the group using teacher-made materials scored significantly higher on arithmetic computation than the other two groups using high cost commercial materials or assorted inexpensive materials.

Comparing the effectiveness of using blocks and ice cream sticks in teaching place value and addition and subtraction algorithm, Knaupp (1970) found that both types of manipulatives produced significant gains in achievement without significant differences although the blocks model seemed to be more enjoyable to the pupils than the stick model.

(c) Some studies tested Dienes' multiple embodiment hypothesis: "...that in mathematical learning abstraction will be more likely to take

place if a multiple embodiment of a mathematical idea is provided, rather than a single embodiment such as Cuisenaire rods by themselves."³

Skipper (1972), in a study with prospective elementary teachers, compared three treatments--one using Dienes blocks and variable base abaci, another using only Dienes blocks, and a third with only lecture presentations. It was concluded that two perceptual embodiments yielded results as good as or better than one perceptual embodiment and that the lecture method yielded results as good as or better than a presentation using the Dienes blocks.

Turek (1972), also in a study with preservice elementary teachers, found no significant differences between a lecture approach and a Dienes-based approach using multiple embodiments, except on one part of the evaluation using manipulatives, the Dienes-based groups performed significantly better.

Sole (1952), in a study to determine whether the use of a variety of materials produces better results than the use of only one materials produces better results than the use of only one material, concluded that if both treatments are used for the same amount of time, then using a variety of materials does not produce better results.

Wheeler (1971) found significant correlations between the number of embodiments that second grade children could manipulate for two-digit addition and subtraction and their performances on multi-digit problems in the symbolic mode, holding age, IQ, and basic fact competence constant.

(d) There were a number of studies comparing the enactive, iconic, and symbolic modes of presentation from Bruner's theory of cognitive growth and theory of representation.

In a study of second graders' thinking in subtraction problems, Gibb (1956) found that problems presented in a semi-concrete context resulted in significantly higher levels of performance than problems presented in a concrete context, and lowest performance levels resulted from the abstract context. Curry (1970), in a study in which clock arithmetic was taught to third graders, found that both the concrete and semi-concrete groups performed significantly better than the abstract group on tests of computation and understanding of principles. Portis (1972), in an analysis of fourth, fifth, and sixth graders performances on problems, found that use of physical and pictorial aids resulted in significantly higher performances than use of symbolic aids. Carmody (1970), in a study comparing symbolic, semi-concrete, and concrete treatments with sixth graders, found that semi-concrete and concrete groups performed significantly higher on transfer tests. The semi-concrete group was also significantly higher than the symbolic group on a numeration test.

³ Some basic processes involved in mathematical learning. In J. M. Scandura (ed.) Research in Mathematics Education. Washington, D. C.: National Council of the Teachers of Mathematics, 1967, 22-23.

In two studies with college students, Archer (1972) and Austin (1974) found significant differences in treatments favoring concrete and semi-concrete over symbolic on most of the tests given.

Some of the studies combined modes of representation or compared only two of the three modes. Devor and Stern (1970), in a study with four-year-olds, compared a picture treatment with an object treatment and a no-treatment control group, finding significant gains in performances, but no significant differences between treatments. In a study comparing iconic and symbolic categorization, Bail (1970) found that first graders showed significantly less preference and significantly less ability for symbol categorization and that there was a transition, with increasing grade level, from the use of the iconic mode to the use of the symbolic mode. Fennema (1972) however, showed that second graders using a symbolic mode to learn multiplication as the union of equivalent disjoint sets performed significantly higher on recall and transfer than those using a concrete treatment. Punn (1973) compared methods of teaching multiplication facts to third graders with manipulatives and symbols, pictures and symbols, or manipulatives, pictures and symbols. The pictorial-symbolic approach yielded significantly lower results in achievement than either concrete-symbolic or the concrete-pictorial-symbolic treatments. Ekman (1966) found that a method to teach addition and subtraction algorithms to third graders using manipulatives produced significantly higher understanding and transfer than methods using pictures or a presentation of the algorithm directly, but no significant differences in skill subtests were found between the treatments. Armstrong (1972), in two studies with trainable mentally retarded and educable mentally retarded, found partial support of the hypothesis that pupils in the beginning stages of representational thought should exhibit greater learning in a concrete mode if the concept requires representational thought than pupils in more advanced stages of representational thought and pupils in more advanced stages learn better when a higher level mode is used.

(e) Other studies compared different kinds of classroom use for a manipulative. Several studies compared a teacher demonstration of the manipulative with individual use of the manipulative. Among those which found that individual manipulation of materials produced significantly superior achievement was a study done by Gilbert (1974). In one of two schools where the study was conducted it was found that students in a group using individual manipulation of the aids scored significantly higher than students in groups using either teacher demonstration or small group manipulation of the aids. However, in the other school no significant differences were found. In a study done by Toney (1968), the data indicated a trend toward greater achievement by the group individually manipulating the materials, although the achievement was not significantly greater than that of the teacher demonstration group. Other studies which showed no significant differences in achievement between teacher demonstration and individual manipulation of the aids were studies done by Jamison (1962), Knaupp (1970), and Pigford (1974).

A study done by Trueblood (1967) found that a teacher demonstration group performed significantly better than an individual manipulation group. In the teacher demonstration group, the pupils observed the demonstration and told the

teacher how to manipulate the visual-tactual aids. However, on a retention test there were no significant differences in groups.

A study by J. E. Smith (1974) compared the effects of two different ways of using the same manipulatives on retention and achievement. One group of first graders was taught two-digit addition using bundled straws with either an adjacent-to-digit method or a juxtaposition method. No significant differences were found.

Harvin (1964) analyzed the effects of frequent or infrequent use of manipulatives on the achievement of classes in 51 elementary schools. A significant difference in mean achievement was found between classes which used manipulatives frequently and those using them less frequently.

(f) Several studies investigated the effects of the use of manipulatives on the attitude of students toward mathematics. Results of a study by Punnett (1973) indicated that treatments using both manipulative materials and mathematical symbols or manipulatives, pictorial devices and symbols produced significantly improved attitudes in pupils, while the attitude declined in pupils in a treatment using only pictorial aids and mathematical symbols. Studies by Harshman, Wells, and Payne (1962), Higgins (1970) and Knaupp (1970) showed no significant differences among attitudes of pupils in varying treatments using different manipulatives or comparing a manipulative with a lecture approach. However, in Knaupp's study there was a non-significant trend toward more independence by the pupils in the student activity class over the students in the teacher demonstration class. Although Sherer (1967) found no significant attitude differences among pupils in an experimental group being tutored with an approach using instructional aids as compared to a group being tutored by traditional methods, more favorable attitudes were found in the pre-service tutors of the experimental group.

Studies that investigated attitudinal changes of pre-service or in-service teachers in classes using manipulative aids generally showed that a manipulative approach significantly increased positive attitude changes. Among these studies were those by Fuson (1975), King (1975), Wall (1972), Warkentin (1975), and Weisman (1972). In the study by Weisman, activity learning produced an attitudinal shift in the positive direction while the traditional approach experienced a negative attitudinal shift.

(g) Many studies investigated differential effects of the use of manipulatives with students having various learner characteristics. These studies respond to the question, "What are the characteristics of the learner who best responds to instruction which uses manipulatives?" They focused in a specific way on the interaction between the learner characteristics and the use of manipulatives. Most of these studies were concerned with finding any significant interactions between ability level (as defined by IQ or performance on an achievement test) and treatment. Studies falling into this category included those by Archer (1972), Curry (1970), Davidson (1972), Harshman, Wells, and Payne (1962),

Jamison (1962), Kuhfittig (1972), Portis (1972), Reddell and DeVault (1960), Trask (1972), and Wallace (1974).

Armstrong (1972) found significant interaction between level of cognitive development and the representational mode of presentation in a study with trainable mentally retarded and in a study with educable mentally retarded in cases of mathematical learning that required representational thought.

Bail (1970), in a study to find interactions between a child's classification operativity and representational mode, concluded that the dependence of cognitive growth on operationality is not to the degree assumed by Piagetian theory.

A study by Devor and Stern (1970) exhibited significant differences showing that four-year-old girls learn more effectively from picture stimuli than objects, whereas for boys there was no significant difference. Wallace (1974) found no significant sex interactions for a multisensory approach or traditional approach in teaching a mathematical concept to fourth, fifth, and sixth graders.

A study by J. E. Smith (1974) found no significant conservation by treatment interaction, but first grade children who were high in conservation of length and area had better retention in addition than those classified as low.

Weber (1970) found no significant interactions between socioeconomic group and manipulative versus paper and pencil treatments, but noted a trend favoring the manipulative treatment for low socioeconomic status children.

Archer, J. A. Effect of concrete, semi-concrete, and abstract teaching methods on mathematical achievement, transfer, and retention at the college level (Doctoral dissertation George Peabody College for Teachers, 1972). Dissertation Abstracts International, 1972, 33, 1580A. (University Microfilms No. 72-25, 370)

Thirty three college freshmen were pretested for mathematical ability, divided into ability levels by means of American College Test composite scores, and randomly assigned to one of three treatment groups for a three-hour study of the function concept. Three teachers each taught each group once. Common lesson plans were used, but these were supplemented by (a) diagrams and drawings for the semi-concrete group, and (b) physical materials for the concrete group. Achievement and transfer tests were administered the day following completion of instruction, and a retention test was given twenty-five days later.

A 3 x 3 factorial design, providing for three levels of ability and three treatment groups, was used. Analysis of variance and orthogonal comparisons were performed for each posttest. No significant differences were found

between students who used aids (concrete and semi-concrete groups combined) and those who did not (abstract group). Significant differences favor the concrete over the semi-concrete group were found at the .10 level on the transfer test and the .05 level on the achievement and retention tests.

Post-hoc examination revealed that the treatment groups differed significantly with respect to ACT composite scores. Using these scores as the covariate, analysis of covariance revealed significant differences at the .10 and .05 levels favoring the concrete over the abstract group on the achievement and transfer tests, respectively. Other post-hoc comparisons at each of the three ability levels revealed that although the differences between the concrete and semi-concrete groups were significant at the medium ability level on all three posttests, the differences were not significant at the low level for any of the tests and were significant at the high level only on the retention test.

Armstrong, J. R. Representational modes as they interact with cognitive development and mathematical concept acquisition of the retarded to promote new mathematical learning. Journal for Research in Mathematics Education, 1972, 3, 43-50.

Two studies were conducted to examine the hypothesis that pupils in the beginning stages of representational thought should exhibit greater learning on mathematical concepts which require representational thought when a concrete, enactive mode is used as the instructional intervention than pupils in more advanced stages of representational thought and conversely, pupils in more advanced stages should better learn concepts requiring representational thought when a higher level model (iconic and/or symbolic) of presentation is used.

Subjects for the first study were 20 Trainable Mentally Retarded (TMR) of mental age 2-4; for the second study 67 Educable Mentally Retarded (EMR) of mental age 5.8-11.9. The EMR subjects were stratified in three levels of mental age, corresponding to stages of representational thought. In both studies pupils were randomly assigned to either manipulative or non-manipulative instructional programs.

The EMR study used a twenty-lesson autoinstructional program of slides, tape, and application packets. The manipulative and nonmanipulative programs differed only with respect to the application packets, one providing physical materials requiring manipulation, the other providing pictures and/or symbols allowing no manipulation. The TMR study was similar in design, although instruction was provided by teachers (systematically rotated between the treatments) rather than machine. The subset relation, numeral-quantity association, conservation of a quantity numeral identification, and counting were among the mathematical concepts presented.

Multivariate analysis of covariance was used, organized by randomized block layout. Pretest subtests and IQ were used as covariates, posttest subtests as variates. To maintain similar power functions for the two studies, levels of confidence were .05 (TMR) and .10 (EMR).

Results partially supported the original hypothesis. Pupils in the early stages of representational thought (TMR study) learned significantly more with manipulation on concepts requiring representational thought than they did without manipulation on concepts not requiring such thought. Pupils in the later stages of representational thought (EMR study) did not respond differentially among the three levels of mental age to mode of instruction and type of concept involved (i.e. there were no interactions), but consistently across cognitive levels as indicated by mental age, mathematical learning which required representational thought was better learned under the manipulative mode.

Austin, J. O. An experimental study of the effects of three instructional methods in basic probability and statistics. Journal for Research in Mathematics Education, 1974, 5, 146-154.

Seventy-one college students (mostly underclassmen not majoring in the sciences) were assigned to one of three treatments. The manipulative-pictorial (MP) treatment used the results of student-performed experiments and graphs, diagrams, and figures in the written material; those in the pictorial (P) treatment performed no experiments, but experimental data was presented to them in the same pictorial forms as used in the MP treatment. The symbolic (S) treatment used material identical to the P material, except that all pictorial aids were removed.

Instruction consisted of twelve lessons, each with behavioral objectives, problems, and a half-hour taped lecture. The oral lectures differed across treatments only when the written lessons differed. Between-student contact was minimized, and no student had direct contact with the instructor.

A posttest yielded a total score and four component subscores (comprehension, computation, application, and analysis) based on the cognitive level assigned to each item of the test. These five scores were subjected to analysis of variance, followed by Sheffe's test for pairwise comparisons of treatment means. All tests were made at the .05 level.

On the total test and on the application and analysis subtests, the symbolic treatment mean was significantly lower than either the MP or P means. On the comprehension subtest, the S mean was significantly lower than the P mean. On the computation subtest, no differences were found. These results tend to confirm that there are risks for the learner when enactive and pictorial methods of instruction are skipped, but that college-level students can give up manipulation of physical objects with no loss in achievement.

Bail, F. T. The relative dominance of ikonic and symbolic categorization in the first, third, and fifth grades (Doctoral dissertation, Cornell University, 1970). Dissertation Abstracts International, 1971, 31, 6392A. (University Microfilms No. 71-14, 614)

Seventy-two children from the first, third, and fifth grades were individually tested. Each was given a Piagetian test of hierarchial classification, designed to measure classification operativity. In the remaining tasks the child was shown cards with a word and a drawing of conflicting meanings on each. As each card was shown, the child was asked to match it with one of four displayed exemplar cards (each with a word and a drawing). For each card shown there were only two logically categorizations: matching the word on the instance card to a semantically related word on one of the four exemplar cards, or the instance card to a semantically related drawing on one of the exemplar cards. After an introductory task, two series of 12 cards were shown; the child could consistently match words, consistently match pictures, or give mixed correct responses, as well as incorrect responses. The first series used eight mutually exclusive categories of common words and common pictures. The second series differed by using less familiar categories of words and pictures.

Inter-grade differences indicated there was a transition, with increasing grade level, from use of the iconic mode to use of the symbolic mode. First graders showed significantly less preference and significantly less ability for symbol categorization.

The experimenter also concluded that the dependence of cognitive growth on operationality is not to the degree assumed by Piagetian theory, since symbol use was more clearly related to grade level than to operative classification measure.

Branch, R. C. The interaction of cognitive style with the instructional variables of sequencing and manipulation to effect achievement of elementary mathematics (Doctoral dissertation, University of Washington, 1973). Dissertation Abstracts International, 1974, 34, 4857A. (University Microfilms No. 74-2244)

Ninety sixth grade students were ranked on Sigel's Cognitive Style Test. The upper and lower 36 were defined as high and low analytic pupils, respectively. Nine high and nine low analytic pupils were randomly assigned to each of four treatments: inductive manipulative, inductive nonmanipulative, deductive manipulative, and deductive manipulative. On four consecutive days, addition and subtraction of positive and negative integers were taught in 25-minute sessions. The next day, immediate posttests of retention and transfer were given; four weeks later, delayed retention and transfer tests were given.

The inductive treatment received examples, with the generalization given at the end of the session; the deductive treatment received the generalization first, then examples. The manipulative treatment pupils had plastic number lines at their desks, the nonmanipulative pupils had nothing.

A 2x2x2 randomized posttest-only factorial design, providing for two levels of cognitive style, two levels of sequencing, and two levels of manipulation, was used. Analysis of variance was performed on each of the four posttest scores.

Pupils taught using the manipulatives scored higher ($p < .05$) on the immediate retention measure than those taught without manipulatives. Inductive sequencing with manipulative use produced higher ($p < .005$) scores on immediate retention than did deductive sequencing without manipulatives. Pupils identified as having low analytic cognitive styles scored higher ($p < .05$) on transfer measures when taught inductively rather than deductively.

Carmody, L. M. A theoretical and experimental investigation into the role of concrete and semi-concrete materials in the teaching of elementary school mathematics (Doctoral dissertation, The Ohio State University, 1970). Dissertation Abstracts International, 1971, 31, 3407A. (University Microfilms No. 70-26,261)

The hypothesis that the use of concrete or semi-concrete aids contributes to the student's organization of mathematical knowledge (his learning of concepts and his ability to apply concepts to new situations) was tested. Three sixth grade classes were randomly assigned to three experimental treatments: symbolic, semi-concrete, and concrete. During eleven class periods taught by the researcher, each group studied topics on number bases, properties of even and odd numbers, and divisibility tests based on the decimal representation of a number. Pretests, posttests, and two transfer tests were administered. In one transfer test students were asked to identify the number base used in certain arithmetic examples. In the other, students were asked to devise tests for divisibility for certain number bases other than base ten.

Results of the tests were analyzed using analysis of covariance, with IQ, mathematics ability, mathematics achievement, and pretest scores used as covariates. In the posttests, the only significant difference ($p = .05$) found favored the semi-concrete group over the symbolic group on the numeration test. Differences at the .01 level were found on the transfer tests, favoring the semi-concrete over the symbolic on both tests and favoring the concrete over the symbolic on one test. No differences were found between the concrete and semi-concrete treatments.

The experiment supported the use of concrete or semi-concrete materials if the goal is transfer.

Churchill, E. M. The number concepts of the young child. Leeds University Research and Studies, 1958, 17, 34-49.

Two groups, each of eight children aged 5 years, were selected after being tested on their number concept with questions similar to those used by Piaget so that the control and experimental groups were matched in their understanding of ideas about number. The experimental treatment consisted of four weeks of play sessions with selected shapes and toys. The play was guided so that the children were led to discover the invariance of number relations. Both groups were tested at the end of the four-week period and again three months later. The experimental group performed significantly better than the control at both times, indicating that the children who had the opportunity to play with materials developed mathematical concepts more quickly than those who did not.

Clausen, T. G. A developmental study of children's responses to multi-sensory approach in mathematical (Doctoral dissertation, University of Southern Mississippi, 1971). Dissertation Abstracts International, 1972, 32, 4830A. (University Microfilms, No. 72-9065).

Eight classes of kindergarten and first grade pupils were used (177 students). Four classes were exposed to a multi-sensory mathematics program, and four classes used the Scott Foresman Worksheet textbook for a six-month period. A mental age was obtained for each child from the Columbia Test of Mental Maturity; the Metropolitan Readiness Test, Level A, was used as a posttest to measure achievement.

Sheffe's t-test was used to compare achievement between the experimental and control groups at each mental age level. (Mental age ranged from four to eight years.) No significant differences were found. Overall, there was a trend toward higher achievement among the pupils exposed to the multi-sensory approach.

Curry, R. D. Arithmetic achievement as a function of concrete, semi-concrete and abstract teaching methods (Doctoral dissertation, George Peabody College for Teachers, 1970). Dissertation Abstracts International, 1971, 31, 4032A-4033A. (University Microfilms No. 71-4258)

Three intact classes of third grade students were randomly assigned to three methods of teaching clock arithmetic. In the concrete method, each child was given clocks to manipulate; in the semi-concrete method, the teacher referred to pictures of clocks; in the abstract method only verbal reference was made to clocks. Each class met for five sessions, studying addition and subtraction on 12-number and 8-number clocks. Since each class was taught by a different instructor, an observation instrument

was developed to identify whether planned methodological differences were followed and to identify other similarities and differences between classes and teachers. Two posttests, one of computational skills, the other testing understanding of principles, were given twice to each student. Aids used during instruction were permitted on the first occasion, but not on the second. In addition, students were separated into high and intermediate ability levels by using composite computation and problem solving scores from the Metropolitan Achievement Test.

Hypotheses were tested by using a three-factor analysis of variance with repeated measures on one factor and by orthogonal comparisons. The results of the observations indicated that planned differences of method were maintained and that other differences were not severe.

No significant method by ability level interactions and no ability level differences were found on any of the posttests. The combined concrete and semi-concrete groups scored higher than the abstract group on all tests except understanding-of-principles without aids. The concrete group scored higher than the semi-concrete group only on the understanding-of-principles test with aids. On this test, the concrete group benefited more from using clocks than did the semi-concrete group from using pictures of clocks.

Davidson, J. E. The impact of selected concrete materials on the understanding of certain mathematical concepts by grade 3 and grade 4 students (Doctoral dissertation, Columbia University, 1972). Dissertation Abstracts International 1973, 33, 6232A. (University Microfilms No. 73-10,915)

Two general hypotheses were investigated: (a) students using concrete materials will show a greater gain in understanding of mathematical concepts and conservation concepts than students taught by more conventional textbook-drill methods, and (b) among students using concrete materials, those with IQ's below their grade median IQ will show greater gain in understanding of mathematical concepts and equal understanding of conservation concepts as those students with IQ's at or above the grade level median.

Each of 432 children in the study was given the Lorge-Thorndike Intelligence Test to establish IQ, the Iowa Test of Educational Achievement (form 4) as a pretest in September, and the Iowa Test (form 3) as a posttest. Piagetian conservation tests were administered to a sample of 160 students at the time of posttesting. During the six-month instruction period children in the experimental groups had all concepts introduced through the use of concrete materials. After achieving understanding of a concept, these children used the adopted textbook as their main tool. The control groups used no concrete materials, but relied on the adopted test and drill materials.

The mean gain in months on the arithmetic concepts portion of the Iowa Test showed no significant differences between students using concrete materials and those not. Among grade 3 children, the experimental lower IQ group had significantly higher scores on the conservation tests than the

corresponding control group (weight and length of the .05 level, mass at the .01 level). Among grade 4 children, the experimental higher IQ group scored higher (.01 level) on the conservation of length test than did the corresponding control group. At this level, concrete materials seemed particularly to enhance the geometry topics in the textbook. No conclusions were drawn with respect to the second general hypothesis.

Devor, G. M. and C. Stern. Objects versus pictures in the instruction of young children. Journal of School Psychology, 1970, 8, 77-81.

Thirty-six four-year-old children were pretested for general ability and assigned on a stratified-random basis to one of three treatments: object stimuli, picture stimuli, and control. The two-day instructional program taught the children verbal labels for the parts of a house and parts of a door and their functions. Object and picture treatments received identical tape-recorded commentary; children in the object treatment had a doll house, door and other objects to use, while children in the picture treatment were shown color drawings made directly from the objects. The control group received no instructions. Each student was given identical pre and posttests: a series of questions was asked in the presence of the objects, then the same questions were asked using pictures of the objects.

Posttest scores were subjected to analysis of covariance, with pretest and general ability scores as covariates. Scores were analyzed by sex and by treatment.

Both experimental groups were superior ($p < .01$) to the control group on the posttest. There was no significant difference between the picture and object treatments. There was a sex by treatment interaction, indicating that girls learn more effectively from picture stimuli than from objects at this age.

Ekman, L. G. A comparison of the effectiveness of different approaches to the teaching of addition and subtraction algorithms in the third grade (Doctoral dissertation, University of Minnesota, 1966). Dissertation Abstracts, 1967, 27, 2275A-2276A. (University Microfilms No. 67-12)

Twenty-seven intact classes from the St. Paul public schools were randomly selected and assigned to one of the treatments. Treatment 1 consisted of presenting algorithms immediately, Treatment 2 developed ideas using pictures before presenting algorithms, Treatment 3 used cardboard disks manipulated by the pupil to develop ideas before presenting algorithms. All treatments used the same pupil worksheets and teacher guidesheets, all teaching was by guided discovery, and the instruction period was 18 days long. A three-scaled test, measuring understanding,

transfer, and computational skill, was administered as a pretest, a posttest, and a 64-week retention test to each pupil.

Three covariance analyses were run on each scale: posttest adjusted for pretest, retention test adjusted for posttest, and retention test adjusted for pretest. Using class as the experimental unit, no significant differences were found among treatments.

Because the varying class size (11 to 33) might mask differences, the data were also analyzed using pupil as the experimental unit. Under this analysis, Treatment 3 produced significantly better ($p=.035$) understanding than Treatments 1 or 2 at the end of the instruction period. The significance of this difference fell to $p=.15$ over the period from pretest to retention test. On the transfer scale, Treatment 3 was superior ($p=.04$) to Treatments 1 and 2 over the entire period. On the skill scale, there was insignificant difference between treatments over the entire period. Several other trends were found.

- Fennema, E. H. The relative effectiveness of a symbolic and a concrete model in learning a selected mathematical principle. Journal for Research in Mathematics Education, 1972, 3, 233-238.

Ninety-five second grade children who measured at or above criterion level on a qualifying exam were randomly assigned to one of eight groups, each of which was then given either a concrete or a symbolic treatment. The topic studied was previously unlearned: multiplication defined as the union of equivalent disjoint sets. The qualifying exam tested for necessary background knowledge. Both treatments learned a symbolic statement of the principle in the general form $\underline{a}, \underline{b} \rightarrow \underline{c}$. The concrete treatment used Cuisenaire rods and modeled $3, 2 \rightarrow 6$ as: 3 two-rods end to end are equivalent in length to a six-rod. In the symbolic treatment, $3, 2 \rightarrow 6$ was modeled as $2 + 2 + 2 = 6$ or "3 twos go with 6 because 2 plus 2 plus 2 equals 6." All other instructional activities, including worksheets, problems, and drill games, were the same for the 14 instructional sessions. One teacher taught all groups.

A test of recall and two transfer tests were given. The transfer tests used ordered pairs having products of 11 . . . 16, while the instruction and recall test were limited to products less than or equal to ten. On the first transfer test, pupils were allowed to use the materials assigned to their treatment. On the second test, given one week later, all pupils were permitted to use counters. All tests had high content validity and reliability. Data were analyzed by one-way analysis of variance, using group means.

High scores on the recall test indicated that both treatments learned the principle to the point of direct recall. Scores favored the symbolic treatment ($p < .090$) over the concrete treatment. Mean scores on the first transfer test revealed the same trend ($p < .053$). On the second transfer test symbolic treatment groups scored significantly ($p < .003$) higher than concrete

treatment groups. Thus either method was effective for direct learning, but the symbolic treatment was more effective when transfer or extension of the learned principle was involved. This may have been due in part to the presymbolic experiences of the children in the study: they had the prerequisite knowledge in their cognitive structures, and most had previous active experience with concrete manipulation; thus they were ready, as suggested by Bruner, to use the symbolic model with its greater generalizability more effectively.

Fuson, K. The effects on preservice elementary teachers of learning mathematics and means of teaching mathematics through the active manipulation of materials. Journal for Research in Mathematics Education, 1975, 6, 51-63.

In this study, the researcher developed a substantial amount of curriculum materials for use with preservice elementary teachers which emphasized the use of manipulatives, created or adopted several instruments and techniques for evaluating teacher learning of such materials, and examined various effects of such teacher learning.

Sixteen Master of Science in Teaching students enrolled in a combined mathematics/mathematics-methods course were the subjects. The course met in 20 sessions of 2½ hours each, in a laboratory setting. The course was designed to teach the content of elementary school mathematics, implicitly presenting a model of how to teach mathematics to children. Most of the topics covered progressed from work with manipulative materials and recording results to analysis of the implications of these results for the symbolic mathematics concepts involved.

The researchers results indicated that students in such a program expressed increased desire to use manipulatives in teaching, increased in at least one aspect in ability to use manipulatives in teaching, and, in fact, did use manipulatives to a great extent in practice teaching. A significant increase in positive attitude toward mathematics was also found after the learning experience with manipulative materials.

Gibb, E. G. Children's thinking in the process of subtraction. Journal of Experimental Education, 1956, 25, 71-80.

Thirty-six second grade children were randomly selected from 24 schools. Each was individually interviewed and asked to solve nine subtraction problems having minuends less than ten. Each problem was presented in one of three applications (take-away, additive-subtraction, or comparative-subtraction) and in one of three contexts (concrete, semi-concrete, or abstract). The concrete context used toys and other objects, the semi-concrete used circles and squares mounted on cards,

the abstract context used verbal problems (about the concrete objects) typed on cards. The nine combinations of application and context were presented to each child.

The interviews were recorded and each response was analyzed with respect to six variables (process, understanding, equation, solution, time, and verbalization). A $3 \times 3 \times 36$ randomized block design with one observation per cell was used in applying analysis of variance techniques to each variable. A composite score was also analyzed using analysis of variance techniques. A confidence level of .01 was used.

Significant differences indicated that the highest degree of attainment was made on take-away problems, the lowest on comparative; additive problems took longer; highest levels of performance were for problems presented in the semi-concrete context, lowest for the abstract context. There were no statistically significant interactions between applications and contexts, although the presence of interactions both between pupils and applications and between pupils and contexts suggests that children conceive of subtraction in various ways and respond differently to varying contexts.

Gilbert, R. K. A comparison of three instructional approaches using manipulative devices in third grade mathematics (Doctoral dissertation, University of Minnesota, 1974). Dissertation Abstracts International, 1975, 35, 5189A. (University Microfilms No. 75-2099)

One hundred twenty-four subjects from two suburban schools received three weeks of instruction in addition and subtraction of two-digit numbers. A prerequisite skills test and identical pre and posttests were given. Student scores were eliminated from the data if they scored below criterion on the prerequisite skills test or above criterion on the pretest. Remaining students were stratified in three ability groups by pretest scores and randomly assigned to one of three treatments: a demonstration (D) treatment in which students observed and advised the teacher on how to manipulate the instructional devices, an individual (I) treatment in which each student was provided with a set of manipulatives, and a group treatment (G) in which groups of four students worked with a set of manipulatives. Teachers were assigned to different treatment groups each week in a balanced rotation. The manipulatives used were counting straws, counters and place value sheets, and abaci.

A $2 \times 3 \times 3$ factorial design, allowing for schools, ability and treatment groups, was used together with analysis of variance on posttest mean scores. There was a significant interaction between schools and treatments. In one school students in treatment I scored significantly higher than students in treatments D or G. There was a directional trend of $I > D > G$. In the other school there were no significant differences. Within the ability levels no consistent pattern of treatment means could be found.

Harshman, H.W., D. W. Wells, and J. N. Payne. Manipulative materials and arithmetic achievement in grade 1. Arithmetic Teacher, 1962, 9, 188-191.

Twenty-six first grade classes, containing 654 pupils, were given one of three year-long treatments: Program A used the commercial materials known as Numberaid, Program B used assorted inexpensive materials, Program C used homemade materials furnished by the teacher. Other differences among the programs were cost of materials (A used high cost materials), content covered (A covered substantially more than is usually taught in first grade, B covered slightly less than A), and amount of in-service training (A received most, B slightly less, C none).

An attitude scale was administered four times during the year and the Stanford Achievement Tests for Arithmetic Reasoning and Computation were given in May. Analysis of variance was applied to both class means and individual scores in attitude and achievement.

Using class means, no significant differences were found. Using individual scores, no significant differences were found between Programs A and B. Using individual scores, differences significant at the .01 level were found in arithmetic computation (in favor of Program C) and total arithmetic achievement in the intelligence subgroup, IQ 100-114 (also in favor of C).

Harvin, V. R. Analysis of the uses of instructional materials by a selected group of teachers of elementary school mathematics (Doctoral dissertation, Indiana University, 1964). Dissertation Abstracts, 1965, 25, 4561A. (University Microfilms No. 65-394).

About 180 teachers from 51 elementary schools in a midwestern city were surveyed and their students were given beginning and end-of-school-year achievement tests.

Teachers classified as frequent users of instructional materials in mathematics tended to have had a teacher preparation course in elementary mathematics instruction. Frequent and infrequent users of aids had taken similar mathematics content courses as undergraduates, and years of teaching experience did not seem related to frequency of use. Teachers of grade 1 indicated they used manipulative materials more than pictorial or symbolic, while teachers in grades 2 - 6 used pictorial and symbolic more often. There was a significant difference in the mean achievement between the classes who used instructional materials frequently and those who used them less frequently.

Higgins, J. L. Attitude changes in a mathematics laboratory utilizing a mathematics-through-science approach. Journal for Research in Mathematics Education, 1970, 1, 43-56.

Twenty-nine eighth grade mathematics classes were taught a four-week SMSG unit, Graphing, Equations, and Linear Functions. During four training sessions, the teachers used the laboratory equipment involved and discussed potential student responses and difficulties. Before and after the instructional period a battery of three achievement and eighteen attitudinal scales from the National Longitudinal Study at Mathematical Abilities was given.

Each pair of pre and posttreatment means was compared using a t statistic for correlated samples. Significant gains ($p < .001$) were found for the three achievement scales. Significant differences were found for six of the attitude scales; for five of these six, posttreatment means were lower than pretreatment, indicating a less favorable attitude toward mathematics after the instructional period.

Using hierarchical grouping analysis on two randomly selected samples of the experimental population, eight "natural" attitude groups were formed such that all the students in a given group had similar attitudes toward mathematics. Differences in attitude between groups were not reflected in significant differences in either ability or achievement. About six percent of the students developed strong, cohesive, unfavorable attitudes while about eight percent changed favorably; most students, however, changed very little in their attitudes toward mathematics.

Jamison, K. W., Jr. The effectiveness of a variable base abacus for teaching counting in numeration systems other than base ten (Doctoral dissertation, George Peabody College for Teachers, 1962). Dissertation Abstracts, 1963, 23, 3816 (University Microfilms No. 63-1882)

Three intact classes of seventh grade students were assigned to one of three treatments. One class received instruction with a large abacus which was demonstrated only by the instructor. Another class had the large abacus plus smaller abaci for each pupil. The third class received only blackboard and chalk instruction. Pre and posttests surrounded five days of instruction.

Individual gain scores were subjected to analysis of variance and no significant differences were found among the three treatments. Further analysis revealed no differences between boys and girls or among low IQ pupils.

King, C. C. Development and evaluation of an activity-based probability unit for prospective elementary teachers incorporating the teaching of mini-lessons to elementary school children (Doctoral dissertation, The Florida State University, 1975). Dissertation Abstracts International, 1976, 36, 5178A. (University Microfilms No. 76-2658)

Forty-eight elementary education majors were paired on results of the Cooperative Arithmetic Test and randomly assigned to experimental and control groups. The experimental group performed six probability activities, the control group used a traditional lecture-text format. Half of each group taught a probability mini-lesson to two elementary school children, the other half did not. The four resulting routines were evaluated on the basis of attitude toward probability and the teaching of probability (pre and post treatment), achievement, and two-week retention.

No significant differences were found among treatments on achievement or retention. There was a trend in favor of the experimental group on an attitude item dealing with knowledge of how to teach probability in the elementary school, but there were no significant differences between the routines with mini-lessons and those without.

Knaupp, J. E. A study of achievement and attitude of second grade students using two modes of instruction and two manipulative models for the numeration system (Doctoral dissertation, University of Illinois at Urbana-Champaign, 1970). Dissertation Abstracts International, 1971, 31, 6471A. (University Microfilms No. 71-14,832)

Four intact classes of second grade students received four weeks of instruction on the addition and subtraction algorithms and place value. Two classes used blocks of wood, two used ice cream sticks. For each manipulative, one class used a teacher-demonstration mode and one used a student-activity mode of instruction. Four attitude tests which utilized sketches of students engaged in various activities of an arithmetic class were developed. Subjects were asked to mark faces showing varying degrees of happiness or sadness to show how they would feel if they were in the scene shown in the picture or, in other tests, to mark which of two pictures they would most like to be in.

All four classes showed significant achievement gains (.01 level). No changes were found in student attitudes toward learning arithmetic, although there was a non-significant trend toward more independence by the student-activity classes. There was a significant and positive relationship between a student's preference for manipulative activities and his preference for teacher assistance.

Kuhfittig, P. K. F. The effectiveness of discovery learning in relation to concrete and abstract teaching methods in mathematics (Doctoral dissertation, George Peabody College for Teachers, 1972). Dissertation Abstracts International, 1972, 33, 1323A. (University Microfilms No. 72-25,393)

Forty seventh grade students were given two class periods of instruction on converting American to old English currency and vice versa. Two methods of teaching were used: intermediate guidance (discovery), involving a structural sequence of questions, and maximal

guidance, involving careful explanations of individual steps. Two levels of learning aids were used: the concrete group was given models of currency to manipulate while only verbal references to currency were made in the abstract group. Students were identified as high or low ability based on a departure of at least one standard deviation from the mean on Metropolitan Achievement Test subscores (middle ability students were excluded from the study).

Identical posttests and four-week retention tests were given for achievement, horizontal transfer, and vertical transfer. No aids were allowed in these tests.

A completely crossed, balanced $2 \times 2 \times 2 \times 2$ factorial design (aids $\times$ guidance $\times$ ability $\times$ tests) with repeated measures on the tests factor was used, together with analysis of variance.

For low ability students, the mean score on posttest achievement was greater for those using concrete aids than for those having no aids. For high ability students, no such difference was found. For intermediate guidance, the mean scores on posttest transfer were higher for those using concrete aids, than for those not. No such difference was found in the maximal guidance group. No significant differences were found between treatments on the retention tests.

Pigford, V. D. A comparison of an individual laboratory method with a group teacher demonstration method in teaching measurement and estimation in metric units to preservice elementary teachers (Doctoral dissertation, The Florida State University, 1974). Dissertation Abstracts International, 1975, 35, 4306A-4307A. (University Microfilms No. 75-941)

Two intact classes of elementary education majors were assigned to two treatments. In the laboratory method 28 subjects handled equipment and participated in measurement and estimation activities. The 29 subjects in the lecture demonstration group met as a class and individually recorded results of the same activities performed by the lecturer. All students covered four units dealing with length, weight, capacity, and temperature. Tasks within units were related to four objectives: selecting appropriate units, reading instruments, converting between units, and estimating quantitative properties of familiar objects.

Similar pre, post and retention tests were administered, involving written multiple choice items and oral short response items given in the presence of the physical materials. Covariate analyses, and t-tests were used in data analysis; an alpha level of .05 was specified.

No differences between the two treatments were found on either the posttest or the retention test. All differences between means of posttest and pretest, retention test and pretest, and retention test and posttest were significant for each treatment. Thus both methods were equally effective.

Portis, T. R. An analysis of the performances of fourth, fifth and sixth grade students on problems involving proportions, three levels of aids and three IQ levels (Doctoral dissertation, Indiana University, 1972). Dissertation Abstracts International, 1972, 33, 5981A. (University Microfilms No. 73-10,853)

One hundred thirty-eight fourth, fifth, and sixth grade children were tested on problems involving proportions. The subjects were ranked by IQ and were separated into high, medium, and low ability groups. Members of each of these ability groups were then assigned randomly to one of three experimental groups. The three experimental groups were given tests according to one of the three levels of aids: physical, pictorial, or symbolic.

Analysis of data showed significant differences among mean performances of grade levels, ability groups, and type of aid. The major conclusions drawn were that the mean performance of sixth grade children was higher than that of fourth and fifth grade children; mean performance of high IQ children was significantly higher than that of middle and low IQ children; mean performances of children on tests using physical and pictorial aids were significantly higher than those on tests using symbolic aids; mean performances of fourth grade children indicated that they did not possess necessary skills for problems involving ratios and proportions; and the modified case study indicated that children solving proportion problems generally use an additive strategy rather than a proportional strategy.

Punn, A. K. The effects of using three modes of representation in teaching multiplication facts on the achievement and attitudes of third grade pupils (Doctoral dissertation, University of Denver, 1973). Dissertation Abstracts International, 1974, 34, 6954A. (University Microfilms No. 74-9739)

Ninety students in three third grade classrooms were taught multiplication facts for 30-40 minutes per day over a nine-week period according to one of three treatments. Treatment 1 used manipulative materials and mathematical symbols; treatment 2 used pictorial devices and mathematical symbols; and treatment 3 used manipulative materials, pictorial devices, and mathematical symbols. Analysis of the data showed that treatments 1 and 3 yielded significantly higher achievement than treatment 2. Also the attitudes of pupils in treatments 1 and 3 improved significantly while the attitudes of pupils in treatment 2 had decreased.

Redell, W. D. and M. V. DeVault. In-service research in arithmetic teaching aids. The Arithmetic Teacher, 1960, 7, 243-246.

Two-hundred-seventy pupils in twenty-four fifth grade classes were given instruction over a five-month period according to one of three treatment groups. Group 1 used the Educator, a hand-operated calculator; group 2 used the Abacounter, a commercially available abacus type aid;

and group 3 used various teacher-made aids (place value chart and number line). Pupils were tested for achievement before and after the treatment and teachers were also tested for arithmetic understanding.

Analysis of the data showed both pupils and teachers of groups 1 and 2 had significantly higher gains in achievement and understanding than group 3.

Sherer, M. T. An investigation of remedial procedures in teaching elementary school mathematics to low achievers. (Doctoral dissertation, the University of Tennessee, 1967). Dissertation Abstracts, 1968, 28, 4031A-4032A. (University Microfilms No. 68-3768)

Forty-seven pupils from grades three through seven were given lessons in elementary mathematics in one of three groups: a control group, receiving only classroom instruction; an experimental group receiving classroom instruction and tutoring lessons from a traditional approach; and a second experimental group receiving classroom instruction and tutoring lessons using instructional aids. The experimental groups were given twenty forty-minute lessons. Pre-service elementary education majors served as tutors. Twelve tutors tutored the sixteen members of the first experimental group and sixteen tutors tutored the sixteen members of the second experimental group. No significant differences in attitude were found among pupils but the tutors of the second experimental group showed significantly more favorable attitudes.

Skipper, S. W. A study of the use of manipulative materials as multiple embodiments for the study of numeration systems by prospective elementary teachers (Doctoral dissertation, University of Missouri-Columbia, 1972). Dissertation Abstracts International, 1973, 34, 1168A. (University Microfilms No. 73-21,485)

One-hundred forty-five prospective elementary teachers were given instructions over four consecutive fifty-minute classes according to one of three treatments: group A used the Dienes multi-base arithmetic blocks and variable base abaci, group B used only the Dienes multi-base arithmetic blocks, and group C had lecture presentations. Three classes were given the treatments one semester and three another semester. A test on numeration systems developed by the experimenter was given in two forms--one as a pre-test and the other as a posttest. Data was analyzed by analysis of covariance with the pre-test scores as covariate.

The results showed no significant differences in performance by the three groups during the first semester. However, in the replication of the experiment the next semester significant differences were found to exist on all three assessments: (1) understanding of basic concepts of positional numeration systems, (2) ability to perform nondecimal computations, and (3) ability to transfer learning to new and similar systems and extensions of the systems studied. Group C had a significantly higher adjusted mean on (1) than group A, and group A had a significantly

higher adjusted group mean on (2) than group B. Also, group C had a higher adjusted group mean on (3) than group A.

The experimenter concluded that the lecture method of presentation yielded as good or better results than a presentation using the ideas of Dienes. The experimenter also concluded that a presentation of positional number systems using two perceptual embodiments yielded results as good as or better than a presentation using only one perceptual embodiment.

Smith, C. W. A comparison of the effect of the activity approach and the lecture approach used in teaching general college mathematics to risk students and the relationship between the two methods and learner characteristics (Doctoral dissertation, Temple University, 1975). Dissertation Abstracts International, 1975, 36, 3476A-3477A. (University Microfilms No. 75-28,299)

Fifty-four college students were given a general mathematics course according to one of two treatments: an activity approach and a lecture approach. One-way analysis of variance was used to test the effects of method of instruction on mathematical achievement and attitude towards mathematics. Three learner characteristics--attitude towards mathematics, locus of control, and need achievement--were correlated with mathematics achievement in a correlation matrix for each of the treatments.

The results showed significant better mathematical performance by the manipulative group, no significant differences in attitude, and no significant relationships between mathematical achievement and the three learner characteristics for either group.

The experimenter concluded that teaching mathematics by the activity approach can be considered as an alternative to the lecture approach.

Smith, G. J. The effects on college general education mathematics students of learning mathematics through the active manipulation of materials (Doctoral dissertation, University of Oregon, 1974). Dissertation Abstracts International, 1974, 35, 809A-810A. (University Microfilms No. 74 - 18,908)

Materials which emphasized a concrete approach to instruction utilizing manipulation of objects, images, and symbols were developed for a general education mathematics course. Subjects for the study were 69 students enrolled in three sections of the course taught by the researcher over a two-year period. The first section used a textbook-lecture approach, the second section served as a pilot for materials, and the third received the completed experimental course. Pre-course attitudes and achievement were measured through previous mathematics course grades, A. C. T. scores, attitude test scores, and the Watson-Glaser Critical Thinking Appraisal. Post-course attitudes and achievement were evaluated

through the Watson-Glaser instrument, a course appraisal questionnaire, questionnaires related to the use of manipulatives, and individual interviews.

Achievement in the three sections seemed generally comparable. Attitude of the students enrolled in the two experimental sections toward the course itself differed significantly from the attitude of students in the lecture section, but attitude of the students toward mathematics in general did not differ between the two instructional programs.

Smith, J. E. The effect of positioning manipulatives in addition with children at two levels of conservation on retention and achievement (Doctoral dissertation, University of Houston, 1974). Dissertation Abstracts International, 1975, 35, 7144A. (University Microfilms No. 75-10,741)

Sixty first-grade children were tested for conservation of length and area, classified as high or low conservers, and randomly assigned to one of two treatments involving five days of instruction on 2-digit addition. In the Adjacent-to-Digit method, children and teacher placed bundled straws representing the tens column to the left of the tens digits and straws for the units columns to the right of the ones digit. In the Juxtaposition method, all materials were placed to the right of the symbols in the problem. All other instructional techniques were identical.

On the fifth day of instruction an achievement test was given, and one week later a retention test was given. Multivariate analysis of covariance was used, with an addition pretest score as a covariate, for two levels of conservation and two levels of treatment.

No significant conservation by treatment interaction was found. There was no significant treatment effect. Children who were high in conservation of length and area had better retention in addition than those classified as low.

Sole, D. The use of materials in the teaching of arithmetic (Doctoral dissertation, Columbia University, 1957). Dissertation Abstracts International, 1957, 17, 1517-1518. (University Microfilms No. 57-2430)

Two-hundred-forty children in twelve classes were taught topics in arithmetic according to two treatments to determine whether the use of a variety of materials produces better results than the use of only one material.

The experimenter concluded that using a variety of materials does not produce better results than using only one material if both treatments are

used for the same time. And if a topic is taught using more time, it will yield better results whether or not multiple embodiments are used.

Spross, P. M. A study of the effect of a tangible and conceptualized presentation of arithmetic on achievement in the fifth and sixth grades (Doctoral dissertation, Michigan State University, 1962). Dissertation Abstracts International, 1962, 23, 1293. (University Microfilm No. 62-4467)

One-hundred sixty-six fifth and sixth graders in 8 classes were taught arithmetic 35 minutes per day over an entire school year according to one of two treatments: (1) a tangible, conceptualized presentation in which concepts were presented at the rate of one per week by the teacher and students reacted by presenting a tangible item to represent their understanding of the concept; and (2) a controlled, routine, self-contained classroom presentation. Standardized achievement tests were given at the beginning, middle, and end of school year and an analysis of covariance was used to analyze data. Significant differences in reasoning were found between the two groups indicating that the tangible and conceptualized presentation of arithmetic is a feasible alternative to the traditional method of classroom presentation.

Toney, J. A. S. The effectiveness of individual manipulation of instructional materials as compared to a teacher demonstration in developing understanding in mathematics (Doctoral dissertation, Indiana University, 1968). Dissertation Abstracts International, 1968, 29, 1831A-1832A. (University Microfilms No. 68-17,296)

Fourth grade students in two classes were taught mathematics over a full semester according to one of two methods: group 1 pupils saw only a teacher demonstration of instructional materials, and group 2 pupils were given the materials to individually manipulate.

Although no statistically significant difference was found between the two class means, the experimenter found that the data indicated a trend toward greater achievement by the group individually manipulating the materials. The experimenter also found the individually manipulated materials to be somewhat more effective in building understanding. However, it was also noted that teacher demonstration of materials is as efficient in promoting mathematical achievement as individual manipulation of materials by the pupils.

Trask, M. W. A study on interaction between aptitudes and concrete vs. symbolic teaching methods as presented to third-grade students in multiplication and division. (Doctoral dissertation,

The University of Oklahoma, 1972). Dissertation Abstracts International, 1973, 33, 4253A. (University Microfilms No. 73-4971)

Two third grade classes were given instruction in multiplication and division according to a symbolic approach (textbook, blackboard, and flash-cards) or a manipulative approach (textbooks plus concrete objects). A regression-type analysis indicated that students of above average ability in arithmetic computation according to the Stanford Achievement Test on arithmetic computation were helped by the manipulative approach, while students of below average computational ability benefited more from symbolic instruction.

The experimenter, therefore, warned against generalization in the area of concrete manipulation since his literature review yielded more cases where below average ability students benefited more from a manipulative than a symbolic approach in contrast to his experimental findings.

Trueblood, C. R. A comparison of two techniques for using visual-tactual devices to teach exponents and non-decimal bases in elementary school mathematics. (Doctoral dissertation, The Pennsylvania State University, 1967). Dissertation Abstracts, 1968, 29, 190A. (University Microfilms No. 8755)

Seven fourth grade classes were taught 21 lessons about exponential notation and non-decimal bases according to one of two treatments: Treatment 1 pupils were able to manipulate visual-tactual aids used in the lessons. Treatment 2 pupils would observe and tell the teacher how to manipulate the visual-tactual aids. An immediate posttest and retention test after four weeks were administered. Analysis of covariance was performed on these scores with mental age as the covariate. Results showed achievement of the observation group performed marginally significantly better ($p=.10$) on achievement than the manipulative group; however pupils taught by the observation method did not retain significantly more than pupils taught with the individual manipulative approach. Achievement test scores correlated highly with mental age and both groups had a high degree of retention.

Turek, S. Application of the multi-embodiment principle in a mathematics methods course for preservice elementary school teachers (Doctoral dissertation, University of Missouri-Columbia, 1972). Dissertation Abstracts International, 1973, 33, 5022A-5023A. (University Microfilm No. 73-7101)

Students in five classes of pre-service elementary school teachers were randomly assigned to four treatment groups to be taught a unit on finite mathematical systems. Two treatment groups were taught by the lecture method and two were taught in small groups using a set of forty-four lessons employing Dienes-based approach with manipulative materials. Equivalent forms of a test developed by the experimenter were used as pre-test and posttest. This test consisted of three parts: Part A dealt with properties of finite mathematical

systems; part B also with finite mathematical systems, but manipulatives were needed to answer the questions; and part C was concerned with infinite mathematical systems.

The experimenter found that there were no main or interaction effects of pre-testing and the method of instruction. An analysis of covariance for part B of the test for treatments during one semester showed a significant difference favoring the Dienes-based approach, but no other significant differences were found.

Wall, C. E. A study of the efficacy of a graduate mathematics methods course in changing in-service elementary teachers' attitudes towards school mathematics (Doctoral dissertation, Michigan State University, 1972). Dissertation Abstracts International, 1973, 33, 6210A. (University Microfilms No. 73-12,846)

Two classes of in-service elementary teachers were taught a method course for a three hour block per week for ten weeks. The purpose of the study was to determine if this course, which involved activity materials, had an effect on the attitudes of in-service teachers towards school mathematics or had an effect on their mathematical understanding. Fourteen variables (sex, degree program, age, years of teaching experience, years of high school mathematics, credits of college mathematics, grade level assignment, mathematics vs. non-mathematics teaching, years at present grade level assignment, perceived attitude, grade level attitude developed, average mathematics grades, attitude, and mathematical understanding) were also checked for pairwise correlation. Pre-tests of attitude and mathematical understanding (form A) were given the first class period; and the attitude test, test of mathematical understanding (form B) and an information questionnaire were given during the last class.

Date analysis indicated there was a significant positive change in the attitude of the in-service elementary teachers and there was no significant change in their mathematical understanding.

Wallace, P. An investigation of the relative effects of teaching a mathematical concept via multisensory models in elementary school mathematics (Doctoral dissertation, Michigan State University, 1974). Dissertation Abstracts International, 1974, 35, 2898B-2899B. (University Microfilms No. 74-27,501)

One-hundred fifty-four fourth, fifth, and sixth graders were taught a mathematical concept over a three-week period according to either a manipulative approach or a traditional approach. A pre-test was given to all pupils and at the end of the three weeks a posttest was administered to all pupils and a manipulative test to fifteen pupils each from the two approaches.

Achievement of the students in the manipulative approach was significantly higher than that of students in the control group on both achievement and manipulative tests. The experimenter concluded from this that the use of multisensory models in learning activities increases meaning and understanding. No significant differences were found for grade level, sex, or welfare status.

The experimenter also makes recommendations that in-service training, pre-service training, the elementary school mathematics curriculum, and school systems all incorporate multisensory approaches to help students learn better.

Warkentin, G. The effect of mathematics instruction using manipulative models on attitude and achievement of prospective teachers.
Journal for Research in Mathematics Education, 1975, 6, 88-94.

Fifteen sections of a theory of arithmetic course for prospective elementary school teachers were given instruction over a one semester period for 50 minutes 3 times a week according to one of two treatments. One-hundred forty-nine students from six sections received instruction utilizing a laboratory approach and structural manipulative models. The control of one-hundred ninety-seven students in nine sections were given instruction in the same topics using a lecture approach and a textbook. Attitudes and mathematical achievement tests were given prior to instruction and at the end of the semester.

An analysis of data showed the experimental group had a significantly (.01 level) more favorable attitude. During the semester the experimental group showed a significant positive change in attitude, but the control group did not. The control sections did, however, perform significantly better on the comprehensive final test, but the laboratory sections were not able to cover as much content as the control sections.

Weber, A. W. Introducing mathematics to first grade children: manipulative vs. paper and pencil (Doctoral dissertation, University of California, Berkeley, 1969). Dissertation Abstracts International, 1970, 30, 3372A. (University Microfilms No. 69-18,873)

Six first grade classes--three of low socioeconomic status and three of middle socioeconomic status--were randomly assigned to one of two treatments for thirty consecutive school days. Treatment group 1 received reinforcement of mathematical concepts through paper and pencil follow-up activities and treatment group 2 received reinforcement through the use of manipulative and concrete materials for follow-up activities.

No significant difference was found between the two treatments as measured by the Metropolitan Readiness Test, which was used as a pre-test and posttest, however a trend did favor the manipulative treatment. On an

experimenter made Oral Test of Understanding, the children from the manipulative treatment scored significantly higher in number of correct responses and in levels of understanding.

No significant differences in achievement were found for socioeconomic groups, but a trend favoring the manipulative treatment for low socioeconomic status children was present.

Weisman, G. L. Mathematics activity learning material for prospective elementary teachers and a comparative study of its application (Doctoral dissertation, The University of Toledo, 1972). Dissertation Abstracts International, 1973, 33, 4255A-4256A. (University Microfilms No. 73-3505)

Forty-five prospective elementary teachers were given one of two treatments for four, sixty-minute periods per week over ten weeks. Treatment 1 was an activity learning section with activity lessons using the textbook as a format for order of presentation. Treatment 2 was a traditional learning section with the usual textbook presentation. Pre-tests and posttests were given examining mathematics achievement, attitude toward mathematics, problem solving ability, and cognitive abilities.

No significant differences in achievement were found for the two methods of instruction, but several attitudinal changes were noted favoring the activity learning section. It was also found that the activity learning section made greater gains in problem solving abilities and cognitive process abilities.

Wheeler, L. E. The relationship of multiple embodiments of the regrouping concept to children's performance in solving multi-digit addition and subtraction examples (Doctoral dissertation, Indiana University, 1971). Dissertation Abstracts International, 1972, 32, 4260A. (University Microfilms No. 72-6737)

One hundred forty-five second grade children were categorized into one of three levels of abstraction by testing them on regrouping two-digit addition and subtraction of four isomorphic concrete embodiments: the abacus, bundling sticks, place-value chart, and multi-based arithmetic blocks. The children were also grouped into three IQ levels. Then they were tested in the symbolic mode on problems involving regrouping two-digit addition, multi-digit addition, two-digit subtraction, and multi-digit subtraction.

The analysis of data showed no significant difference between the means on levels of abstraction and performance on two-digit problems in the symbolic mode. However, children across the IQ levels who were proficient in regrouping on multiple embodiments scored significantly higher on the multi-digit problems than children who were not proficient using the concrete materials. Significant correlations were also found between the number of embodiments that children were able to manipulate for two-digit addition and subtraction and their performances on multi-digit problems in the symbolic mode while holding age, IQ, and basic number fact competence constant.

II. Content-oriented research

Some of the research on manipulatives was topic or content-oriented. The main thrust of these investigations was to determine the role of manipulatives or the relative effectiveness of a manipulative or non-manipulative approach on the learning of a specific topic at a specific grade level.

In a study with three and four year olds, Williams (1969) found that environmentally deprived children were able to learn basic concepts in linear measurement using a manipulative learning aid that he designed. The aid enabled the child to see in concrete terms the size and space relations used in linear measurement concepts of one-half, one-fourth, and one-eighth.

Pan balances and mathematical balances were used to instruct first grade pupils in the basic addition facts in a study of Dashiell and Yawkey (1974). Comparing these two manipulatives, they found the mathematical balance to yield significantly better results on a standardized arithmetic test on addition facts.

Steffe and Johnson (1970) investigated the problem-solving abilities of first graders. They gave a 48-item test and compared performances of children who were allowed free use of manipulatives in solving the problems with children who had no manipulatives available. They found that the group with manipulative aids performed significantly better than the deprived group on all but one of the eight problem types in all but one of the four ability groups.

In another study with first graders, Prindeville (1971) gave 24 supplementary lessons on place value, order of numbers to 400, and two-place addition and subtraction. The two classes using manipulative materials and language training significantly outperformed the control group in the posttest and retention test.

DeFlandre (1974) field tested a unit on place value numeration with second, third, and fourth grade classes. He concluded that this unit, using manipulative aids and multiple embodiments, was an effective way to teach place value numeration systems and express these abstractions in symbolic form. He also concluded that children who studied this unit could not transfer the concept of place value numeration systems to addition and subtraction at a symbolic level.

In a study involving the recall of basic multiplication facts, Babb (1975) compared three treatments given to second graders: a textbook approach, a manipulative material approach, and an imagery-mnemonic approach. He found the adjusted mean recall score for manipulatives to be significantly higher than that for imagery, but no significant differences in final recall and comprehension were found between the manipulative and textbook approaches, although the manipulative approach yielded a significantly higher attitude score.

In another study of instructing third graders in multiplication using four different treatments, Moody, Abell, and Bausell (1971) found no significant differences in any of the four treatments: activity-oriented, rote, rote-word problem, and control. The validity of this study was strongly questioned by Holz (1972) in his critique of the article reporting their study.

Nichols (1971) compared two methods of instruction in multiplication and division with third graders. She found significant differences favoring a manipulative approach with pupil discovery over a semi-concrete, abstract approach in all the 16 hypotheses of the study.

Several studies involved various approaches to the teaching of fractions. In a study by Brown (1972), four approaches to teaching equivalent fractions to fourth grade pupils yielded the results that a textbook approach was inferior to three other approaches: the textbook with film, textbook with manipulatives, and textbooks with film and manipulatives. He also found the textbook-film-manipulative approach to yield significantly higher mean scores than the other three groups.

Bisio (1970) investigated the instruction in addition and subtraction of like fractions in grade five using three treatments: no manipulatives, teacher-demonstrated manipulatives, and teacher and student use of manipulatives. He concluded that the demonstration use of manipulative materials appeared as effective as use by the student and was better than non-use of the manipulatives.

Green (1969) compared the effects of two instructional approaches and two instructional materials on the teaching of multiplication of fractions with fifth graders. She concluded that diagrams and manipulative aids were equally effective in the learning of multiplication of fractions, and an attitude test showed that pupils like the diagram approach better than the manipulative approach.

Purser (1973) investigated the relation of manipulative activities, achievement, and retention in teaching fractions and decimals to seventh grade pupils. He found the group using manipulative activities had significantly higher scores on posttest and retention tests than the group using only paper-pencil type activities. Bledsoe, Purser, and Frantz (1974) reported similar results in the Journal for Research in Mathematics Education the following year.

Coltharp (1968) compared the effectiveness of a concrete and an abstract approach in teaching integer arithmetic to sixth graders. He found no significant differences and concluded that pupils taught addition and subtraction of integers by an abstract, algebraic approach achieved as well as those taught by a concrete, visual approach.

Bring (1971) investigated the effects of varying concrete activities on the achievement of fifth and sixth graders in learning topics of geometry. He found that instruction using concrete activities produced higher achievement, higher interest, and lower anxiety than instruction without concrete activities.

Babb, J. H. The effects of textbook instruction, manipulatives and imagery on recall of the basic multiplication facts (Doctoral dissertation, University of South Florida, 1975). Dissertation Abstracts International, 1975, 36, 4378A. (University Microfilms No. 76-1638)

Three second grade classes, totalling 76 children, were instructed in multiplication using three treatments. Treatment T, a textbook approach, using multiplication instruction from the Holt School Mathematics third grade text. Treatment M, a manipulative approach, was designed by the investigator with concrete materials available to the child at all times. Treatment I, an imagery-mnemonic approach, used a substitution system to code the digits 0 through 9 into sounds and a set of pictures to recall the multiplication facts. Treatments were conducted for 23, forty-five minute periods with the investigator instructing all classes. Analyses covariates were used to test the hypotheses of the study.

The investigator found that the adjusted mean recall score for manipulatives was significantly higher than that for imagery. A significant difference for occasion by treatment interaction was also found, indicating treatment growth curve interaction. No significant differences were found in final recall and comprehension mean scores, but it was found that the adjusted mean score for attitude was significantly higher for manipulatives than for textbook instruction.

The investigator recommends the use of more manipulatives in the early-elementary classrooms and further research on the use of imagery.

Bisio, R.M. Effect of manipulative materials on understanding operations with fractions in Grade V (Doctoral dissertation, University of California, Berkeley, 1970). Dissertation Abstracts International, 1971, 32, 833A. (University Microfilms No. 71-20, 746)

Twenty-nine fifth grade classes were instructed in addition and subtraction of like fractions using three treatments. In Treatment A, neither teachers nor students used any manipulative materials. In Treatment B, the teacher used manipulative materials as a demonstration for the students. In Treatment C, both teachers and students used manipulative materials. A pretest and posttest of addition and subtraction of like fractions was designed by the experimenter and was given to all students as well as the California Test of Mental Maturity and the Stanford Achievement Test. Analyses of covariance and t-test were used to test for significant differences between the three treatments groups. The method

the student used to arrive at his answers was also determined by the interviewer and this was also recorded.

Children taught with methods B and C were at least equal on measures to children taught by method A and all indications were that Treatment C created no unfavorable results. It was also concluded that the demonstration use of manipulative materials appeared as effective as actual use and was better than non-use of manipulative materials.

Bledsoe, J. C., J. D. Purser, and N. R. Frantz. Effects of manipulative activities on arithmetic achievement and retention. Psychological Reports, 1974, 35, 247-252.

Three-hundred thirty-nine seventh grade students in 12 ability-grouped mathematics classes (2 each of high, medium, and low achievers) were instructed in fractions and decimals according to two treatments. Treatment 1, the experimental treatment, consisted of a series of learning packages on fractions and decimals followed by a series using manipulative activities in measuring objects with a scale and micrometer. Treatment 2, the control treatment, consisted of the identical series of learning packages followed by a series of pencil-and-paper problems on measuring objects. Students worked at their own pace with high ability students finishing in about four days, medium in six days, and low in eight days. A factorial, fully crossed treatment-by-level-by-six analysis of covariance was carried out with the covariate being the Basic Skills in Arithmetic Test pretest. The dependent variable for both achievement and retention tests was the performance on the Basic Skills in Arithmetic Test (posttest given as they completed the units and retention test given four weeks after the posttest).

The experimenters found that the experimental Treatment 1 produced significantly higher results on the posttest and retention tests indicating that manipulatives significantly enhance achievement and retention in seventh grade students. They also found a significant sex difference on the retention test favoring girls.

Bring, C. R. Effects of varying concrete activities on the achievement of objectives in metric and non-metric geometry by students of grades five and six (Doctoral dissertation, University of Northern Colorado, 1971). Dissertation Abstracts International, 1972, 32, 3775A. (University Microfilms No. 72-3248)

One-hundred-two fifth and sixth graders, divided into four classes, were given a two-week long, semi-programmed unit on volume, congruence, symmetry, and isometries of an equilateral triangle and square using two different treatments. One treatment group was given a supplement of concrete materials and activities; the deprived group did not have this supplement, and, where possible, pictures were replaced by verbal descriptions. Two posttests were given--one at the end of the unit and another a week later as a retention test. Using a full regression model with the variables

of treatment group, age, sex, ethnic background, pretest score, reading level score, mathematics level score, and general intelligence, predictions of achievement on the posttest were made. Although scores were higher on posttest I, no significant difference was found. But on posttest II, the retention test, there was a significant difference. The experimenter concluded that concrete activities produced higher achievement and also found anxiety lower and interest higher among students using concrete activities. Further interesting results regarding the other variables were also found.

Brown, C. K. A study of four approaches to teaching equivalent fractions to fourth grade pupils (Doctoral dissertation, University of California, Los Angeles, 1972). Dissertation Abstracts International, 1973, 33, 5465A. (University Microfilms No. 73-10, 410)

Twelve fourth grade classes, totalling 220 pupils, were taught a unit on equivalent fractions over 18 class periods. Twelve different teachers taught the 12 classes. Four groups of three classes each were formed, each group using a different instructional approach. Group A used the state textbook only, group B used the state textbook and a film, group C used the state textbook and manipulatives based on the film, and group D used the textbook, film, and manipulatives. The mean scores for groups B, C, and D were significantly higher than the mean scores for group A. The gains for groups B, C, and D were also significantly greater than group A; in fact, group D performed significantly higher than all the other groups, the increase in performances of groups B and C being about the same.

Coltharp, F. L. A comparison of the effectiveness of an abstract and a concrete approach in teaching of integers to sixth grade students (Doctoral dissertation, Oklahoma State University, 1968). Dissertation Abstracts International, 1969, 30, 924A. (University Microfilms No. 69-14, 237)

Four sixth grade classes, totalling 79 students, were instructed in the addition and subtraction of integers using two methods of instruction. The concrete, or visual, approach used the Greater Cleveland Mathematics program material, which used the number line and other visual aids to develop understanding of integer arithmetic. The abstract, or algebraic, approach was developed by the experimenter and used the ordered pair approach. The two groups were equated using arithmetic achievement scores and IQ scores.

The t-test showed no significant differences in the achievement of the two groups, leading the experimenter to conclude that sixth grade students taught addition and subtraction of signed numbers using an abstract, algebraic approach achieved as well as those taught using a concrete, visual approach.

Dashiell, W. H., and T. D. Yawkey. Using pan and mathematics balances with young children. The Arithmetic Teacher, 1974, 21, 61-65.

Forty first grade children were given initial instruction in the basic addition facts of single digit addends using two approaches over 20 consecutive school days. The approach using pan balances consisted of four phases: free play, balancing through trial and error, instruction in finding sums and solutions to number sentences on worksheets, and solving problems and then explaining their solutions. A standardized arithmetic test on addition facts was given as a pretest and posttest. The experimenters found that the children using the mathematical balance solved a significantly greater number of problems correctly than the children using the pan balance. Also the girls significantly outperformed the boys on both types of balance.

DeFlandre, C. The development of a unit of study on place value numeration systems, grades two, three, and four (Doctoral dissertation, Temple University, 1974). Dissertation Abstracts International, 1974, 35, 6434A. (University Microfilms No. 74-19, 747)

The experimenter field tested a self-constructed unit on place value numeration systems with second, third, and fourth grade classes. The unit, presented over nine weeks, consisted of 27 activity cards developing the structure of place value numeration systems, leading from the manipulation of concrete objects to the process of symbolization. On the basis of this field test, the experimenter formed several conclusions, among which were that the second grade children needed more experiences in manipulative materials than did third and fourth grade children, and that children were unable to transfer the concept of place value numeration systems to addition and subtraction at symbolic levels as a result of their experience with the unit. But he concluded that fourth graders could abstract concepts of the structure of place value numeration systems and could express these abstractions in symbolic form.

Green, G. A. A comparison of two approaches, area and finding a part of, and two instructional materials, diagrams and manipulative aides, on multiplication of fractional numbers in grade five (Doctoral dissertation, The University of Michigan, 1969). Dissertation Abstracts International, 1970, 31, 676A. (University Microfilms No. 70-14, 533)

About 480 fifth grade pupils with five teachers were taught a 12-day unit on multiplication of fractional numbers using two approaches: area and finding a part of, and two instructional materials: diagrams and manipulative aids. Daily criterion tests, posttest, retention test, attitude tests, and transfer tests were given. Analyses of variance and covariance were used to analyze data with the Californic Arithmetic achievement test as covariate. Although the diagrams groups achieved significantly higher than the materials group on Test 6 total score, posttest total score part 1--concepts, because the materials group had higher actual and adjusted means on applications, and there were no other significant differences, the experimenter concluded that diagrams and materials were equally effective in learning multiplication of fractions. It was also concluded that when comparing the four treatments, the area-diagram approach was the best, with

the of-materials group ranking second. The attitude test showed pupils liked using diagrams significantly better than using manipulative materials.

Holz, A. W. Comments on the effect of activity-oriented instruction. Journal for Research in Mathematics Education, 1972, 3, 183-185.

This article is a critique of the Moody, Abell, Bausell study which found no significant differences of an activity-oriented approach over a rote-treatment and a rote-word problem treatment. Holz raises a number of questions concerning the validity of the study for testing an activity-oriented approach to instruction. He questions their theoretical justification, their testing procedures, and validity of the measures employed.

Moody, W. B., R. Abell, and R. B. Bausell. The effect of activity-oriented instruction upon original learning, transfer, and retention. Journal for Research in Mathematics Education, 1971, 2, 207-212.

Ninety third grade pupils were given instruction in multiplication using four different treatments: activity-oriented, in which the pupils started each day with a class activity requiring manipulation of prescribed materials in prescribed ways; a rote treatment, in which the pupils used a multiplication unit from a textbook; a rote-word problem treatment, in which the same text was used with additional practice in solving word problems; and a control group which received instruction in addition only to ascertain whether learning, transfer, and retention were functions of instruction. The treatments covered four weeks and were preceded by a pretest, posttest, a transfer test four days after the last day of instruction, and retention tests after six and eight weeks. The experimenters used three orthogonal comparisons between treatment means on each of the four sets of scores, and they found no significant differences between the activity-oriented treatment and the two rote and rote-word problem treatments.

Nichols, E. J. A comparison of two methods of instruction in multiplication and division for third-grade pupils (Doctoral dissertation, University of California, Los Angeles, 1970). Dissertation Abstracts International, 1972, 32, 6011A. (University Microfilms No. 72-13, 636)

Two hundred sixty-seven third grade pupils in ten classes were given instruction on the multiplication and division combinations of six, seven, and eight over fifteen 45-minute class periods using one of two treatments. Treatment A used manipulative materials and pupil discovery. Treatment B used semi-concrete and abstract materials. Analysis of covariance was used over fifty-nine variables. Sixteen hypotheses were formed concerning differences in posttest mean gains, differences in durability of mean gains between groups, differences in posttest mean gains for subjects with an IQ of 105 or lower, and differences in durability of mean gains for subjects with an IQ of 105 or lower; each of these four concerns was stated for each of the four areas of: skill in computation in multiplication, skill of computation in division, favorable attitudes toward arithmetic, and under-

standing of arithmetic. Significant differences were found for all 16 hypotheses and the experimenter concluded the use of manipulative materials and pupil discovery were more effective for this population than the use of semi-concrete and abstract materials.

Prindeville, A. C. A program for teaching selected mathematics concepts to first-grade children using manipulanda, language training and the tutor-tutee relationship (Doctoral dissertation, University of California, Los Angeles, 1971). Dissertation Abstracts International, 1972, 32, 6111A. (University Microfilms No. 72-13, 641)

Pupils in three first grade classes were given 24 supplementary lessons on place value, order of numbers to 400, and two-place addition and subtraction during a daily 25-minute enrichment lesson. One class was instructed using the usual large-group setting, and another in small groups by sixth grade tutors. Both of these classes used manipulative materials designed along the Dienes and Cuisenaire models, and descriptive statements were introduced, as part of each lesson, as advance organizers and mediators for each concrete and symbolic operation. The third class was given standard supplementary practice from the workbook and drill sheets accompanying the text used in the class.

Eight analyses of covariance were used with sex of subject and pretest knowledge as covariates. On the final posttest and on the retention test both experimental groups performed significantly better than the control group. It was predicted that the tutorial group would also show greater mastery than the other experimental group, but there was no significant difference. The experimenter concluded that for the population, a manipulanda-language program produced greater mastery than a standard workbook and drill sheet practice on the topics covered.

Purser, J. D. The relation of manipulative activities, achievement and retention, in a seventh-grade mathematics class: an exploratory study (Doctoral dissertation, University of Georgia, 1973). Dissertation Abstracts International, 1973, 34, 3255A. (University Microfilms No. 73-31, 942)

Three hundred forty-six seventh graders in 15 classes were given four units of learning packages on fractions, decimals, using a rule, and using a micrometer according to one of two treatments. The experimental group used manipulative activities and the control group used only paper-pencil type activities. The fifteen classes had been divided into three ability groups --high, medium, and low--at the beginning of the school term.

The experimental group performed significantly better than the control group on both posttest scores and retention scores. Girls also scored significantly better on the retention test than did boys, even though there were no significant differences on the posttest. The experimenter concluded that more manipulative activities should be included in the mathematics program of seventh graders.

Steffe, L. P., and D. C. Johnson. Problem solving performances of first grade children. Athens, Georgia: University of Georgia, Research and Development Center in Educational Stimulation, March, 1970. (ERIC Document Reproduction Service No. ED 041 623)

One hundred and eight first grade children in four ability groups were given a 48-item test in solving arithmetical word problems under two distinct experimental conditions over a one-month period. Half the children were allowed free use of manipulative materials in solving the problems, and half were not given the opportunity to use manipulatives. The four ability groups were: low quantitative comparison scores and low IQ scores, high quantitative comparison scores and low IQ scores, low quantitative comparison scores and high IQ scores, and high quantitative comparison scores and high IQ scores. Children with middle quantitative comparison and middle IQ scores were excluded from the study. The 48 items were six each of the type $a + b = n$, $a - b = n$, $a + n = b$, and $n + a = b$, with each type involving an action, and not involving an action.

Multivariate, univariate, and discriminant analyses were performed on the data. It was found that the group which had manipulative aids available for free use performed significantly better than the group with no manipulatives available on seven of the eight problem types. Only on the problem type $a + b = n$ with objects present were the performances not significantly higher for the higher IQ group.

Williams, M. A. Concept development in measurement at the nursery school level, with a new manipulative learning aid (Doctoral dissertation, Colorado State College, 1969). Dissertation Abstracts International, 1970, 30, 4166A-4167A. (University Microfilms No. 70-7177)

Twenty-one nursery school children were given an eight school day exposure to a manipulative learning aid to test the concepts: one-half, one-fourth, one-eighth, as used in linear measurement. A pretest, posttest, and transfer test relating to the kindergarten rule were given. All but three children showed increases in achievement from pretest to posttest. The experimenter concluded that three-year-old and four-year-old environmentally deprived children used in the study were able to learn basic concepts of linear measurement using the manipulative aid that he designed to enable the child to see size and space relationships in concrete terms. The experimenter cited twenty-eight recommendations for further studies.

III. Catalogs of manipulatives

The purpose of a number of studies, papers, and other endeavors was to catalog manipulative aids available for the teaching of various concepts in elementary school mathematics. Several of these--Adkins (1958), Davidson (1968), Flint (1971), Higgins and Sachs (1974), Shoemaker and Swadener (1972)--are concerned with classifying and listing large numbers of actual manipulatives available or constructable, listing addresses of suppliers from which the manipulatives may be obtained.

Others--Hamilton (1966) and Vest (1969 and 1971)--are concerned more with describing general categories and listing the characteristics of different types of manipulatives. These may or may not suggest or list specific manipulatives.

Kansky (1969) and Williams (1961) give analyses of models and manipulatives, categorizing a small number of manipulatives and paying particular attention to the theory and systems to which the models belong. Williams (1962) gives a very comprehensive view of several specific systems based on manipulatives, stressing the importance of the theoretical basis and technique for use of the particular apparatus in the teaching of various concepts.

Green (1970) surveyed elementary grade teachers across the United States to determine the availability, use, and purpose of 59 specific instructional materials used in teaching mathematics to culturally disadvantaged children.

Adkins, B. E. A topical listing and explanation of selected instructional aids in arithmetic (Doctoral dissertation, State University of Iowa, 1958). Dissertation Abstracts, 1959, 19, 1609.

This listing of instructional aids is accompanied by a bibliography of 234 entries. The eleven topics used to classify the instructional aids were: counting devices, place value devices, recreational aids, casting out nines, instructional games, basic processes, the model store, manipulative arithmetic, games for use outside the classroom, arithmetic bulletin boards, commercial arithmetic games. In the interest of providing motivation for the pupils' use of these aids, the author also includes an historical development of the various types of aids.

Davidson, P. S. An annotated bibliography of suggested manipulative devices. Arithmetic Teacher, 1968, 15, 509-524.

This bibliography contains descriptions of 197 manipulative materials categorized into fifteen general categories: blocks, calculators/computers, cards, construction, drawing tools, geoboards, measuring devices, miscellaneous items, models, numerical games, puzzles, shapes and tiles, strategy games, student instructional materials, and teacher resource materials. Accompanying the brief descriptions of the materials and their uses are prices and addresses of the suppliers at the end of the bibliography. Many

of the materials were developed in England and suppliers there have also been listed.

Flint, D. Supplement to district math guide 1970-71. Fountain Valley School District, California, 1971. (ERIC Document Reproduction Service No. ED 013 371)

This listing of manipulative aids, audio-visual materials, and demonstration materials for the elementary classroom is categorized into nine general categories: numbers and numerals, geometry, measurements, applications, statistics and probability, sets, functions and graphs, logic, and problem solving. Addresses of the suppliers along with catalog numbers for individual items are given.

Green, R. W. A survey of the mathematical instructional materials used in teaching culturally disadvantaged children grades 1 through 6 throughout the United States (Doctoral dissertation, Indiana University, 1969). Dissertation Abstracts International, 1970, 31, 1101A. (University Microfilms No. 70-11, 670)

The purpose of this survey was to determine the availability, use, and purpose of instructional materials used in the teaching of mathematics to culturally disadvantaged children in elementary schools. Fifteen schools of eight randomly selected cities in five geographical sections of the United States were canvassed. The questionnaire was sent to 232 teachers of grades 1 through 6 in these schools and listed 59 instructional materials. Teachers were asked if the materials were furnished by the schools, were teacher made, or were not available. They were also asked to rank their use of a five point scale from never to frequently used and were asked if the purpose of the use was demonstration, student manipulation, display, or other.

Data was analyzed and conclusions were made concerning each of the headings by grade level, geographical area, etc. The general conclusions were: (1) schools furnished very few materials for teaching mathematics to culturally disadvantaged children, (2) schools tended to furnish more materials to lower grades than to upper grades, (3) the amount of materials used in teaching mathematics to culturally disadvantaged children varied from section to section, but varied little from grade to grade, (4) teachers of the first five grades used materials more for student manipulation, while sixth grade teachers used materials more for demonstration.

Hamilton, E. W. Manipulative devices. Arithmetic Teacher, 1966, 13, 461-467.

The author lists seven characteristics which he thinks that good manipulative devices should have. He also lists three characteristics that concepts should have in order to be represented by manipulative aids. The use of the abacus with place value is discussed in some detail to exemplify these characteristics. He then lists 38 references under the headings: manipulative devices, visual aids, concrete materials. These references are largely from The Arithmetic Teacher, each usually presenting one manipulative

device and its use or a general overview of manipulatives.

Higgins, J. L., and L. M. Sachs. Mathematics laboratories: 150 activities and games for elementary schools. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1974. (ERIC Document Reproduction Service No. ED 104 720)

This collection of activities and games for use in elementary school mathematics laboratories is presented in eight categories: number concepts, skills review, measurement, fractions, graphs and functions, and geometric concepts. These activities and games were submitted by classroom teachers and were selected by the authors for their use of manipulative materials or their reliance on student interaction. Each subject area is given a short introduction describing the topics covered in that category. Each activity description contains the goals and purposes of the activity, the materials needed, the procedure for carrying out the activity, and the source from which the activity was obtained.

Kansky, R. J. An analysis of models used in Australia, Canada, Europe, and the United States to provide an understanding of addition and multiplication over the natural numbers (Doctoral dissertation, University of Illinois, 1969). Dissertation Abstracts International, 1969, 30, 1074A-1975A. (University Microfilms No. 69-15, 330)

The purpose of the study was to suggest changes in the use of specific models and to suggest research studies regarding the use of concrete materials in the teaching of arithmetic.

The author examined 254 elementary school mathematics textbooks from the United States and 21 foreign countries in order to identify structural models in use in the teaching of addition and multiplication of natural numbers. He classified the specific models into four categories: counter, rod, geometric, storyline. The models were also classified in the level of abstractness of the materials and the mathematical subsystem of the natural number system which the model paralleled. The teaching effectiveness of each general model was analyzed using ten mathematical criteria and nine pedagogical criteria. Particular attention was given to the theoretical bases for the models.

Shoemaker, T., and M. Swadener. Ideas for manipulative materials--elementary mathematics concepts. Boulder, Colorado: Northern Colorado Educational Board of Cooperative Services, 1972. (ERIC Document Reproduction Service No. ED 064 177)

This collection of activities is a set of cards containing ideas for manipulative materials and activities. The cards are indexed into seven categories: sets, number-numerals, geometry, measurement, probability, number theory, and function. Objectives, supplies needed, and suggested activities are listed on each of the cards.

Vest, F. R. A catalog of models for the operations of addition and subtraction of whole numbers. Educational Studies in Mathematics, 1969, 2, 59-68.

The author describes twelve families of models for addition and subtraction: set-union, decomposition, comparison, machine-type, rod, number-line, structured pattern, counting, operator, McLennan and Dewey, Minnemath, and inverse operation models. Each description contains an example of the family of models. He suggests uses for the models and suggests further research into various strategies for use of the models.

Vest, F. R. A catalog of models for multiplication and division of whole numbers. Educational Studies in Mathematics, 1971, 3, 220-228.

The author describes eleven families of models for multiplication and division of whole numbers: set-union, decomposition, number-line, Cartesian product, rectangular array, comparison, rod, machine-type, two-dimensional, repeated addition and repeated subtraction, and counting models. Included with the description of each family is one or more examples of models which belong to the family. In the summary he suggests other interpretations of multiplication and division from which additional families of models may be developed.

Williams, J. D. Teaching arithmetic by concrete analogy--I. miming devices. Educational Research, 1961, 3, 112-125, 195-213.

In this article the author makes a comprehensive classification and analysis of concrete devices used in the teaching of elementary school mathematics. He classifies manipulatives into and discusses the following nine categories: the abacus, number lengths, accessories for containing and measuring number lengths, number patterns, the pegboard, fraction devices, devices for illustrating powers, some devices for teaching elementary algebra and geometry, and some additional materials. Within each of these categories he discusses thoroughly the subcategories exemplified by different manipulatives. He also includes a comparative summary in chart form of ten devices over twelve categories of classification.

Williams, J. D. Teaching arithmetic by concrete analogy--II. structural systems. Educational Research, 1962, 4, 163-191.

In this article the author makes a very comprehensive summary of eight structural systems used in the teaching of arithmetic and elementary mathematics. The eight structural systems described are: Stern, Cuisenaire, Shaw (with "Structa" apparatus), Jones (with "Avon" apparatus), Montessori, Dienes (multibase arithmetic blocks and algebraic experience materials), Lowenfeld, and Bass. For each of these systems, the apparatus, theory, and technique for various concepts are discussed in detail. Finally, some comparisons of the different systems are made and the age range of and literature on the various systems are discussed.

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